

UNIVERSITY OF TORONTO  
DEPARTMENT OF ECONOMICS

ECONOMICS 381H1F – 2018

**MANAGERIAL ECONOMICS II: PERSONNEL ECONOMICS**

**Midterm  
Version A**

**Instructions**

The test is 50 minutes long. Non-programmable calculators are allowed. The test consists of four questions, each worth 5 points. Show all your work in the space provided below the question. If you need additional space, you may write on the back of the page.

**LAST NAME** \_\_\_\_\_  
**FIRST NAME** \_\_\_\_\_  
**STUDENT NUMBER** \_\_\_\_\_

Good luck!

Question 1	Question 2	Question 3	Question 4	Total
/5	/5	/5	/5	/20

1. A waiter can serve  $q$  customers for each hour of work  $e$ . Assume there are no other factors influencing the number of customers. The waiter's disutility of work is given by  $0.5e^2/k$ , where  $k$  is a positive number that represents years of work experience. The waiter's outside option is 1, while the restaurant owner's outside option is 0. Assume that the owner can observe and verify waiter's hours of work.
  - a. (1 point) If the owner decides to hire the waiter, how many hours should he require the waiter to work?
  - b. (2 points) Show that it is efficient that the owner hires the waiter only if the waiter has at least two years of experience.
  - c. (2 points) Show that the owner must offer a higher salary to attract waiters with more work experience.
  
- a. (1 point) The efficient number of hours of work equates the marginal benefit and marginal cost. The benefit is equal to  $q=e$ , so the marginal benefit is 1. The cost is equal to  $0.5e^2/k$ , so the marginal cost is  $e/k$ . Therefore, the efficient level of  $e$  solves  $1=e^*/k$ , which implies that  $e^*=k$ .
- b. (2 points) It is efficient that the owner hires the waiter if the social surplus evaluated at the efficient level of hours of work is positive, or  $SS(e^*)=q(e^*)-c(e^*)-R-S \geq 0$ . Given that  $e^*=k$  from part (a) and  $R=1$  and  $S=0$ , the social surplus can be expressed as  $SS(e^*)=e^*-0.5e^{*2}/k -1-0=k-0.5k^2/k-1=k-0.5k-1=0.5k-1$ . Therefore,  $SS(e^*) \geq 0$  if and only if  $0.5k-1 \geq 0$ , or  $k \geq 2$ .
- c. (2 points) The minimum salary that the owner must offer the waiter is given by  $w^*=R+c(e^*)$ . Given that  $R=1$  and  $c(e^*)=0.5e^{*2}/k=0.5k$  from part (a), we have that  $w^*=1+0.5k$ . Therefore,  $\partial w^*/\partial k=0.5 > 0$  and the owner must offer a higher salary to attract waiters with more work experience.

2. The Ministry of Health conducted a randomized experiment to test whether physicians paid by the piece rate system are more productive than physicians paid by salary. The results were summarized using the following regression model:

$$E[Y] = \begin{array}{c} 2.5 \\ (0.5) \end{array} + \begin{array}{c} 0.5 \times \text{Piece Rate} \\ (0.1) \end{array}$$

where  $Y$  represents the number of services, Piece Rate is an indicator equal to 1 if the physician is paid by piece rate and 0 if the physician is paid by salary, and the numbers in parentheses represent standard errors.

- a. (0.5 point) What is the average number of services provided by the piece rate physicians?
  - b. (0.5 point) Do piece rate physicians provide significantly more services than salary physicians (in statistical sense)?
  - c. (2 points) Is this evidence consistent with economic theory? Discuss both the case when the physician effort can be observed and the case when the physician effort cannot be observed.
  - d. (2 points) Is this evidence similar in magnitude to the results that Bruce Shearer found for tree planters in British Columbia?
- a. (0.5 point) The average number of services provided by piece rate physicians is 3 services.
  - b. (0.5 point) The piece rate physicians provide on average 0.5 more services than salary physicians. The t-ratio for this estimate equals 5 ( $=0.5/0.1$ ), suggesting that this difference is statistically significant.
  - c. (2 points) When the physician effort can be observed, the level of effort will be identical under both piece rate and salary models and we should observe no difference in productivity. In contrast, when the physician effort cannot be observed, the piece rate physicians are likely to be more productive than the salary physicians. This conclusion follows because physician will likely work more when his pay is tied to his performance, as in the piece rate model, than when his pay is independent of his performance, as in the salary model.
  - d. (2 points) The evidence implies that piece rate physicians provide on average 0.5 more services relative to salary physicians who provide on average 2.5 services. In percentage terms, this is a difference of about 20 percent ( $0.5/2.5$ ). This result is similar to Shearer who found that tree planters paid by piece rate are about 20 percent more productive than tree planters paid by salary.

3. A school principal considers hiring a new English teacher. For each unit of teacher's effort  $e$ , the students' reading score increases by  $q=e+u$ , where  $u$  is a random variable with a mean of 0 and a variance of 1. The teacher's cost of effort is  $c(e)=0.5e^2$  and his outside option is 0. The teacher is risk-averse, with the coefficient of absolute risk aversion equal to 3. The school principal is risk-neutral and her outside option is 0. Assume that the teacher's pay is linear in  $q$  (i.e.  $w=a+bq$ ).
- a. (1 point) Find the expected payoffs for the school principal and for the teacher ( $E[V]$  and  $E[U]$ ) as functions of the teacher's effort  $e$  and the payment contract parameters  $a$  and  $b$ . Note: you do not need to find the optimal contract in this part of the question, just express the expected payoffs in terms of  $a, b$ , and  $e$ .
  - b. (2 points) What is the expected reading score if the teacher's effort can be observed?
  - c. (2 points) What is the expected reading score if the teacher's effort can't be observed and the school principal designs the contract?
- a. (1 point) The school principal's expected payoff is  $E[V]=E[q-w]-0.5s\text{Var}[q-w]=e(1-b)-a-0.5s(1-b)^2\theta=e(1-b)-a$  since  $s=0$ . The teacher's expected payoff is equal to  $E[U]=E[w]-0.5r\text{Var}[w]-c(e)=a+be-0.5rb^2\theta-0.5e^2=a+be-0.5(3)b^2(1)-0.5e^2=a+be-1.5b^2-0.5e^2$ .
  - b. (2 points) If the teacher's effort can be observed, the optimal effort level satisfies the expected marginal benefit equals marginal cost condition. In this question, this implies that  $e^*=1$ . Since  $E[q]=e$ , this also implies that  $E[q]=1$ .
  - c. (2 points) When the teacher's effort can't be observed, the school principal will design a contract to maximize his expected payoff subject to the participation and incentive compatibility constraints. We have  $E[V]=e(1-b)-a$ . Further, the PC constraint states that  $E[U]=R$ , or  $a+be-1.5b^2-0.5e^2=0$ . We also have that the IC constraint implies that  $\partial E[U]/\partial e=0$ , or  $b=e$ . Substituting PC and IC constraints into  $E[V]$ , we have that  $E[V]=b-0.5b^2-1.5b^2$ . The first-order condition for  $b$  is then  $1-b-3b=0$ , or  $b=1/4=0.25$ . From IC constraint, we also have that  $e=b$ . Therefore,  $E[q]=e=0.25$ .

4. Consider the principal-agent relationship described in the following table:

Agent's action	Benefit to the principal	Cost to the agent
Dance	x	\$2
Sing	\$8	\$4

The principal's outside option is zero. Assume that the agent's action can be observed and verified.

- (1 point) If the agent designs the contract, and decides to sing, how much should he charge the principal?
- (2 point) What value of x will make dancing the efficient level of action relative to singing?
- (2 point) Assume that  $x=5$  and that the agent designs the contract. What is the agent's maximum outside option so that it is efficient to form this relationship?

- (1 point) \$8 since it is the maximum he can charge given that the principal's outside option is zero.
- (2 point) For singing, the net social benefit is \$4. For dancing, it is  $x - \$2$ . Therefore, for dancing to be more efficient, it must be the case that  $x - \$2 > \$4$ , or that  $x > \$6$ .
- (2 point) If  $x=5$ , it is efficient that the agent sings. In this case, the agent can charge the principal \$8, so her net benefit will be  $\$8 - \$4 = \$4$ . This has to be larger than the agent's outside option for the contract to be acceptable to the agent. Therefore, if the outside option is greater than \$4, it will not be efficient to form the relationship.